

# New limits on effective Majorana neutrino masses from rare kaon decays

K. Zuber

*Lehrstuhl für Experimentelle Physik IV, Universität Dortmund, Otto-Hahn Str.4,  
44221 Dortmund, Germany*

PACS: 13.15,13.20Eb,14.60.Pq,14.60.St

---

## Abstract

The rare kaon decay  $K^+ \rightarrow \pi^- \mu^+ \mu^+$  is violating lepton number by two units and is investigated within the context of Majorana neutrino masses. Using a new upper bound from E865 an upper limit on the effective Majorana mass term  $\langle m_{\mu\mu} \rangle \lesssim 500$  GeV could be obtained. This is improving existing bounds by about one order of magnitude. Implications on heavy neutrinos are discussed as well as future possibilities to improve limits on this as well as other elements of the Majorana mass matrix are investigated.

*Key words:* massive neutrinos, double beta decay, lepton number violation, rare kaon decays

---

## 1 Introduction

Over the last years growing evidence arose for a non-vanishing neutrino rest mass. The observations of missing solar neutrinos, a deficit in upward going atmospheric  $\nu_\mu$  and the LSND accelerator experiment results all can be explained within the context of neutrino oscillations. For recent reviews see [1,2]. On the other hand oscillation experiments are no absolute mass measurements, depending on  $\Delta m^2 = m_2^2 - m_1^2$ , where  $m_{1,2}$  are the two mass eigenvalues. Therefore several neutrino mass models exist to describe the observed effects. Beside that also the fundamental character of the neutrino, whether being a Dirac or Majorana particle, is still unknown. The most promising channel to probe this property for  $\nu_e$  is neutrinoless double beta decay. The measured quantity  $\langle m_{ee} \rangle$  is called effective Majorana neutrino mass and given by

$$\langle m_{ee} \rangle = \left| \sum U_{ei}^2 m_i \eta_i^{\text{CP}} \right| \quad (1)$$

where  $m_i$  are the mass eigenvalues,  $\eta_i^{\text{CP}} = \pm 1$  are the relative CP-phases and  $U_{ei}$  are the mixing matrix elements connecting weak eigenstates with mass eigenstates. The current experimental upper bound on  $\langle m_{ee} \rangle$  is around 0.2 eV [3]. But this quantity is only one element of a general  $3 \times 3$  matrix of effective Majorana neutrino masses given in the form

$$\langle m_{\alpha\beta} \rangle = \left| \sum U_{\alpha i} U_{\beta i} m_i \eta_i^{\text{CP}} \right| \text{ with } \alpha, \beta = e, \mu, \tau. \quad (2)$$

The limits for the other matrix elements are rather poor compared with double beta decay. Limits on  $\langle m_{e\mu} \rangle$  arise from muon-positron conversion on titanium coming from the experimental bound of [4]

$$\frac{\Gamma(\text{Ti} + \mu^- \rightarrow \text{Ca}^{GS} + e^+)}{\Gamma(\text{Ti} + \mu^- \rightarrow \text{Sc} + \nu_\mu)} < 1.7 \cdot 10^{-12} \quad (90\% \text{CL}) \quad (3)$$

which can be converted in a new limit of  $\langle m_{e\mu} \rangle < 17$  (82) MeV depending on whether the proton pairs in the final state are in a spin singlet or triplet state and allowing correction factors of the order one for the difference in Ti and S as given in [5]. Recently an improved limit on the element  $\langle m_{\mu\mu} \rangle \lesssim 10^4$  GeV was given by investigating trimuon production in neutrino - nucleon scattering [6]. The first full matrix of limits on  $\langle m_{\alpha\beta} \rangle$ , including for the first time matrix elements containing the  $\tau$  - sector of Eq. 2 are given in [7], using HERA charged current results with associated dimuon production. All limits obtained for the matrix elements are of the order  $10^4$  GeV or slightly below. Indirect bounds coming from FCNC processes like  $\mu \rightarrow e\gamma, \tau \rightarrow \mu\gamma$  could also account for severe limits. On the other hand note, that these processes depend on  $m_{e\mu} = \sqrt{\sum U_{ei} U_{\mu i} m_i^2}$  (in case of  $\mu \rightarrow e\gamma$ ) while the ones mentioned before typically depend on  $\langle m_{\alpha\beta} \rangle^2$ . Therefore without specifying a mixing and mass scheme, the quantities are rather difficult to compare. A discussion of such models within this context and using oscillation data can be found in [8]. The same argument holds for combining  $\langle m_{ee} \rangle$  and  $\langle m_{e\mu} \rangle$  to determine  $\langle m_{\mu\mu} \rangle$ . Therefore any experimentally obtained limit is very useful.

A further interesting topic within this context is the production of neutral heavy leptons and direct production of Majorana neutrinos heavier than 100 GeV, the last has been studied for various collider types [9]. The current limits on neutral heavy leptons are coming from LEP and are 39.5 GeV (stable) [10] and 76.0 GeV (for an unstable Majorana neutrino coupling to muons) [11]. Also mixing of such heavy particles with the light neutrinos will be restricted by the limits given for  $\langle m_{\alpha\beta} \rangle$ . The process of a lepton-number violating rare kaon decay discussed in this paper will allow to obtain a new limit on  $\langle m_{\mu\mu} \rangle$ .

## 2 The decay $K^+ \rightarrow \pi^- \mu^+ \mu^+$

A further possibility to probe  $\langle m_{\mu\mu} \rangle$  is the rare kaon decay

$$K^+ \rightarrow \pi^- \mu^+ \mu^+ . \quad (4)$$

This process is violating lepton number by two units. The measured quantity  $\langle m_{\mu\mu} \rangle$  is given by

$$\langle m_{\mu\mu} \rangle = \left| \sum_i U_{\mu i}^2 m_i \eta_i^{\text{CP}} \right| . \quad (5)$$

The lowest order Feynman diagrams are shown in Fig. 1. The amplitude  $A_1$  is given for the tree diagram by [12]

$$A_1 = 2G_F^2 f_K f_\pi (V_{ud} V_{us})^* \sum_i (U_{li} U_{l'i})^* p_{K,\alpha} p_{\pi,\beta} [L_i^{\alpha\beta}(p_l, p_{l'}) - \delta_{ll'} L_i^{\alpha\beta}(p_{l'}, p_l)] \quad (6)$$

where

$$L_i^{\alpha\beta}(p_l, p_{l'}) = m_{\nu_i} (q^2 - m_{\nu_i}^2)^{-1} \bar{v}(p_l) \gamma^\alpha \gamma^\beta P_R v^c(p_{l'}) \quad (7)$$

and  $q$  corresponds to the four-momentum of the virtual  $\nu_i$  and  $P_R = (1 + \gamma_5)/2$ . The box diagram cannot be calculated easily because the hadronic matrix element

$$\int d^4x d^4y e^{i(p_d - p_u)y} e^{i(p_s - p_u)x} \langle \pi^- | [\bar{d}_L(y) \gamma_\beta u_L(y)] [\bar{s}_L(x) \gamma_\alpha u_L(x)] | K^+ \rangle \quad (8)$$

is not directly related to measured quantities like  $\langle 0 | \bar{s}_L \gamma_\alpha u_L | K^+ \rangle$  and  $\langle \pi^- | \bar{d}_L \gamma_\beta u_L | 0 \rangle$  as in the tree graph. The tree diagram is dominating the total decay rate and the  $m_\nu$  dependence is coming basically from  $L^{\alpha\beta}$ . Detailed calculations can be found in [12,13]. Because we are far away from the expected rate, the uncertainties in the matrix element can be neglected. A first extraction of a branching ratio for this process was done in [12] reexamining the data from [14]. They obtained a branching ratio of

$$\frac{\Gamma(K^+ \rightarrow \pi^- \mu^+ \mu^+)}{\Gamma(K^+ \rightarrow \text{all})} < 1.5 \cdot 10^{-4} \quad (90\% \text{CL}) \quad (9)$$

Using this value with the theoretical calculations of [5] a limit of  $\langle m_{\mu\mu} \rangle < 1.1 \cdot 10^5$  GeV could be deduced [15]. The processes discussed in [7] were able to improve that number down to be less than  $4 \cdot 10^3$  GeV. In the meantime

new sensitive kaon experiments are online and using the E865 experiment at BNL a new upper limit on the branching ratio of

$$\frac{\Gamma(K^+ \rightarrow \pi^- \mu^+ \mu^+)}{\Gamma(K^+ \rightarrow \text{all})} < 3 \cdot 10^{-9} \quad (90\% \text{CL}) \quad (10)$$

could be obtained [16], an improvement by a factor 50000. Because the branching ratio is  $\propto \langle m_{\mu\mu} \rangle^2$  this can be converted in a limit on  $\langle m_{\mu\mu} \rangle \lesssim 500$  GeV, a factor of eight better than the existing limits and three orders of magnitude better in this particular decay channel.

### 3 Results and Discussion

The obtained upper limit on  $\langle m_{\mu\mu} \rangle$  of 500 GeV restricts regions of heavy Majorana neutrinos having a mixture  $U_{\mu H}$  with  $\nu_\mu$ . No direct bound exists for neutrino masses heavier than 90 GeV. This is illustrated in Fig. 2. Further improvements to this bound could come from even more sensitive searches for this rare kaon decay. An improvement by a factor of about 10 on  $\langle m_{\mu\mu} \rangle$  implying an improvement on the branching ratio limit by two orders of magnitude would bring the number in overlap with LEP searches. New experiments like E949 at BNL and CKM (E905) at Fermilab [17] or a muon collider could improve on that significantly. Furthermore the decay of charmed mesons could be considered as well. Among the Cabibbo angle favoured modes are  $D^+ \rightarrow K^- \mu^+ \mu^+$ ,  $D_S^+ \rightarrow K^- \mu^+ \mu^+$  or  $D_S^+ \rightarrow \pi^- \mu^+ \mu^+$ . The existing limits on the branching ratio for these processes are  $3.2 \cdot 10^{-4}$ ,  $5.9 \cdot 10^{-4}$  and  $4.3 \cdot 10^{-4}$  respectively [18]. While being competitive with the old bound for the kaon decay discussed, the new kaon branching ratio limit is now five orders of magnitude better. Therefore, to obtain new information on  $\langle m_{\mu\mu} \rangle$  from D-decays, analyses of new data sets have to be done.

To improve significantly towards lighter neutrino masses ( $\langle m_{\mu\mu} \rangle \lesssim 1$  GeV) one might consider other processes. The close analogon to double beta decay and therefore a measurement of  $\langle m_{\mu\mu} \rangle$  using nuclear scales would be muon capture by nuclei with a  $\mu^+$  in the final state as discussed in [19]. No such experiment was performed yet. The ratio with respect to muon capture can be given for  $^{44}\text{Ti}$  and light neutrino exchange as

$$\Gamma = \frac{\Gamma(\mu^- + \text{Ti} \rightarrow \mu^+ + \text{Ca})}{\Gamma(\mu^- + \text{Ti} \rightarrow \nu_\mu + \text{Sc})} \simeq 5 \cdot 10^{-24} \left( \frac{\langle m_{\mu\mu} \rangle}{250 \text{ keV}} \right)^2 . \quad (11)$$

Assuming that a branching ratio of the order of muon-positron conversion (Eq.3) can be obtained, a bound on  $\langle m_{\mu\mu} \rangle \lesssim 150$  GeV results. Unfortunately this is only a slight improvement on the bound obtained here. Furthermore

assuming heavy neutrino exchange for the  $\mu^+$  capture would result in a rate another four orders of magnitude lower than for light neutrino exchange. Improvements on the  $\tau$  - sector of matrix elements of Eq. (2), especially  $\langle m_{\tau\tau} \rangle$ , could be done by a search for rare B-decays. Limits on the branching ratio for decays  $B^+ \rightarrow K^- \mu^+ \mu^+$ ,  $B^+ \rightarrow \pi^- \mu^+ \mu^+$  of less than  $9.1 \cdot 10^{-3}$  exist [20], however nobody looked into the decays  $B^+ \rightarrow K^- \tau^+ \tau^+$  or  $B^+ \rightarrow \pi^- \tau^+ \tau^+$ . With the new B-factories such a search might be possible at a level of producing limits on  $\langle m_{\tau\tau} \rangle$  competitive with the ones given in [7].

## 4 Conclusion

Whether neutrinos are Dirac or Majorana particles is still an open question. For several flavours an effective Majorana mass matrix can be assumed, whose best explored element is  $\langle m_{ee} \rangle$  due to neutrinoless double beta decay searches. An improved limit on  $\langle m_{e\mu} \rangle$  is given here. An investigation especially on the matrix element  $\langle m_{\mu\mu} \rangle$  was performed. Using new bounds on the branching ratio of  $K^+ \rightarrow \pi^- \mu^+ \mu^+$  obtained by E865 a new upper limit on  $\langle m_{\mu\mu} \rangle$  of less than 500 GeV was obtained, improving existing bounds by roughly one order of magnitude. Informations on the admixture of heavy Majorana neutrinos with muons are obtained. Suggestions for further improvements are given as well as the proposal to consider a search for rare B - decays like  $B^+ \rightarrow \pi^- \tau^+ \tau^+$  to improve on the  $\tau$  - sector of  $\langle m_{\alpha\beta} \rangle$ .

## 5 Acknowledgements

I would like to thank H. Ma for providing me the new E865 limit and W. Rodejohann for useful discussions.

## References

- [1] K. Zuber, *Phys. Rep.* **305**, 295 (1998)
- [2] S. M. Bilenky, C. Giunti, W. Grimus, *Prog. Nucl. Part. Phys.* **43**, 1 (1999)
- [3] L. Baudis et al., *Phys. Rev. Lett.* **83**, 411 (1999)
- [4] J. Kaulard et al., *Phys. Lett.* **B 422**, 334 (1998)
- [5] M. Doi, T. Kotani, E. Takasugi, *Prog. Theo. Phys. Suppl.* **83**, 1 (1985)
- [6] M. Flanz, W. Rodejohann, K. Zuber, preprint hep-ph/9907203

- [7] M. Flanz, W. Rodejohann, K. Zuber, *Phys. Lett.* **B 473**, 324 (2000)
- [8] W. Rodejohann, preprint hep-ph/0003149
- [9] W. Buchmüller, C. Greub, *Nucl. Phys.* **B 363**, 345 (1988), *Nucl. Phys.* **B 381**, 109 (1992); J. Gluza, M. Zralek, *Phys. Rev.* **D 48**, 5093 (1993), A. Hoefer, L. M. Sehgal, *Phys. Rev.* **D 54**, 1944 (1996), G. Cvetic, C. S. Kim, *Phys. Lett.* **B 461**, 248 (1999)
- [10] P. Abreu et al., Delphi-coll., *Z. Phys.* **C 74**, 57 (1997)
- [11] G. Abbiendi et al., OPAL-coll., preprint hep-ex/0001056
- [12] L. S. Littenberg, R. E. Shrock, *Phys. Rev. Lett.* **68**, 443 (1992)
- [13] A. Halprin, P. Minkowski, H. Primakoff, S. P. Rosen, *Phys. Rev.* **D 13**, 2567 (1976), J. N. Ng, A. N. Kamal, *Phys. Rev.* **D 18**, 3412 (1978), J. Abad, J. G. Esteve, A. F. Pachero, *Phys. Rev.* **D 30**, 1488 (1984)
- [14] C. Chang et al., *Phys. Rev. Lett.* **20**, 510 (1968)
- [15] H. Nishiura, K. Matsuda, T. Fukuyama, *Mod. Phys. Lett.* **A 14**, 433 (1999)
- [16] H. Ma, E865-coll., private communication
- [17] S. Kettell, preprint hep-ex/0002011
- [18] K. Kodama et al., E653-coll., *Phys. Lett.* **B 345**, 85 (1995)
- [19] J. H. Missimer, R. N. Mohapatra, N. C. Mukhopadhyay, *Phys. Rev.* **D 50**, 2067 (1994)
- [20] A. Weir et al., Mark2-coll. *Phys. Rev.* **D 41**, 1384 (1990)

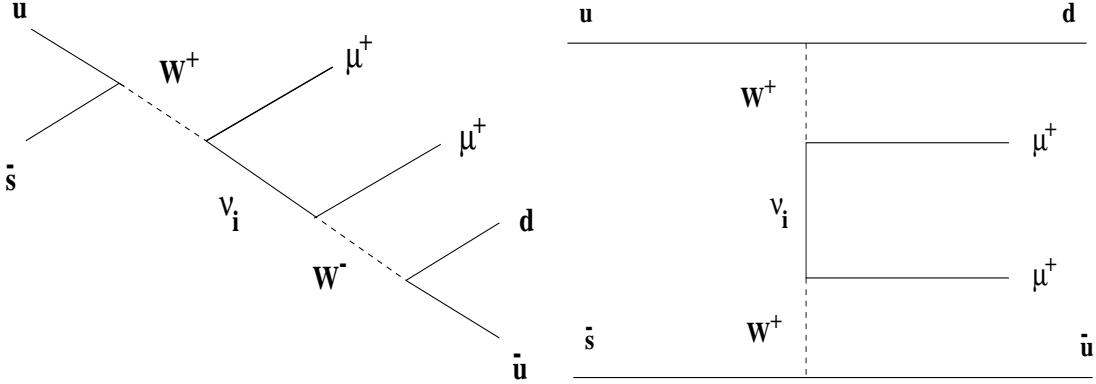


Fig. 1. Feynman diagrams in lowest order contributing to the rare kaon decay  $K^+ \rightarrow \pi^- \mu^+ \mu^+$ . Shown are the tree (left) and box diagram (right).

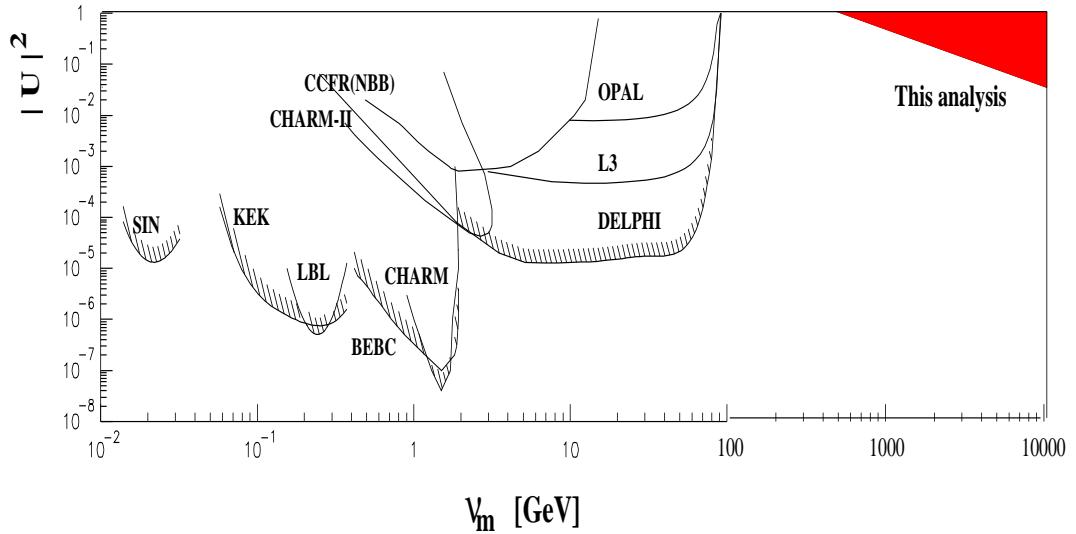


Fig. 2. 95 % CL Limits on the mixing matrix element  $|U^2|$  of a heavy neutrino to a light one. The compilation of limits below 90 GeV are taken from [10]. No direct limit exist for the mixing matrix element  $|U_{\mu H}|^2$  above 90 GeV. The region which can be excluded by the analysis of rare kaon decays is shown in the upper right.